Dimension:	Chapter:	Module:
• A: From STEM to SMART	• Fibers Yarns	• Mathematics

Scope module

Fibers and yarns represent the basic elements of textile fabrics: main aim of this module is to highlight some applications of mathematics for fibers and yarns, meant for smart textiles.

Contents / agenda

1. Physical quantities and mathematical relations for fibers and yarns
2. The modality to introduce physical quantities and mathematical relations
3. The meaning of physical quantities for smart textiles materials

Introduction module

This module includes specific relations for properties of textile fibers and yarns, an explanatory part on how physical quantities are theoretically introduced and the corresponding mathematic formulae, as well as their meaning for smart textiles [1-3].

1. Physical quantities and mathematical relations for fibers and yarns

Fibers and yarns are basic elements for textile fabrics and they are characterized by certain physical properties [1]. Fibers are the building blocks of textiles, they are significantly longer than they are wide. Textile fibers are made of cotton, wool, polyester, or other raw materials, and are typically 1 cm or longer and 10 to 25 micron wide, though a lot of variation is possible. Yarns are long continuous lengths of interlocked fibers, suitable for use in the production of fabrics [2].

Main physical properties of fibers:
- Linear density;
- Hygroscopicity;
- Length;
- Breaking force and elongation at break;
- Breaking tenacity.

Main physical properties of yarns:
- Linear density;

- Hygroscopicity;
- Twist;
- Breaking force and elongation at break;
- Breaking tenacity.

The physical properties of textile fibers and yarns are highlighted by their physical quantities and mathematical formulae [1].

1.1 Linear density

Linear density (or yarn count, yarn number and yarn size) is the expression of the ‘fineness’ of a fiber or yarn and cannot be expressed in terms of diameter because its diameter is not stable and uniform along its length, and its cross-sectional shape may not be circular [3]. Consequently, it is expressed either by measuring the mass of a known length of yarn or by measuring the length of a known mass of yarn [4]. These two basic methods of expression for linear density of any textile yarn are known as:

●the direct system (mass/length)

●the indirect system (length/mass)

Direct System

The direct system is based on measuring the weight per unit length. The main systems in use are:

• T_{ex} - weight in grams of 1000 meters of a fiber or yarn.

$$T_{ex} = \frac{M\ (g)}{L\ (1000m)} = A \cdot \rho \quad (1)$$

Where:
M - represents weight in grams;
L – length;
A – mean value of transversal cross-section of the fibers;
ρ – density.

• Denier - weight in grams of 9000 meters of a fiber or yarn.

$$D_{en} = \frac{M\ (g)}{L(9000m)} = 9 \cdot Tex \quad (2)$$

Indirect system

In the indirect system, fineness N_m is expressed as the ratio of the number of standard lengths per unit mass. A higher designated number indicates a finer fiber or yarn.

Metric count – N_m indicates the number of fiber or yarn meter within a gram. It is defined by following relation:

$$N_m = \frac{L\ (m)}{M\ (g)} = \frac{L}{V \cdot \rho} = \frac{L}{A \cdot L \cdot \rho} = \frac{1}{A \cdot \rho} \tag{3}$$

Where:
V - represents the volume of the fibers of sample;
A – mean value of transversal cross-section of the fibers;
ρ – density.

Between direct system and indirect system there are following relations:

$$Nm \cdot Tex = 1000;\ Nm \cdot Den = 9000 \tag{4}$$

English count – N_e indicates number of hanks of conventional length within a Libra of material (453,59 g) and is defined by the relation:

$$N_e = \frac{L\ [hanks]}{M\ [libra]} \tag{5}$$

Conventional length of a hank, expressed in yards (1 yard = 0,9144 m), depends on fiber type, as follows:

- For cotton fibers and yarns — 1 hank = 840 yds = 768,1 m (6)
- For worsted wool fibers and yarns — 1 hank = 560 yds = 512,06 m
- For woolen fibers and yarns — 1 hank = 256 yds = 234,08 m
- For linen fibers and yarns — 1 hank = 300 yds = 274,32 m

French count – N_f indicates the number of kilometers of fiber and yarn within 500 grams of material and is defined by following relation:

$$N_f = \frac{L\ [km]}{M[0,5\ kg]} \tag{7}$$

Relations for conversion of indicated indexes are presented in table 1.

Table 1 – Conversion table for measurement units

Index	Symbol	Ttex	Den	Nm	Ne	Nf
Count (tex)	Ttex	1	9Ttex	1000/Ttex	590,7/Ttex	500/Ttex
Count (den)	Den	Den/9	1	9000/Den	5315/Den	4500/Den
Metric umber	Nm	1000/Nm	9000/Nm	1	0,59Nm	0,5 Nm
English number (cotton)	Ne	590,7/Ne	5315/Ne	1,69 Ne	1	0,847 Ne
French number	Nf	500/Nf	4500/Nf	2Nf	1,18 Nf	1

1.2 Textile fiber or yarn diameter

Fiber diameter is a parameter characterizing the fiber out of a dimensional point of view and represents the distance measured between the margins visualized on an optical microscope screen [5-8]. Textile fiber and yarn diameter computing is accomplished in spreadsheet software by statistic methods.

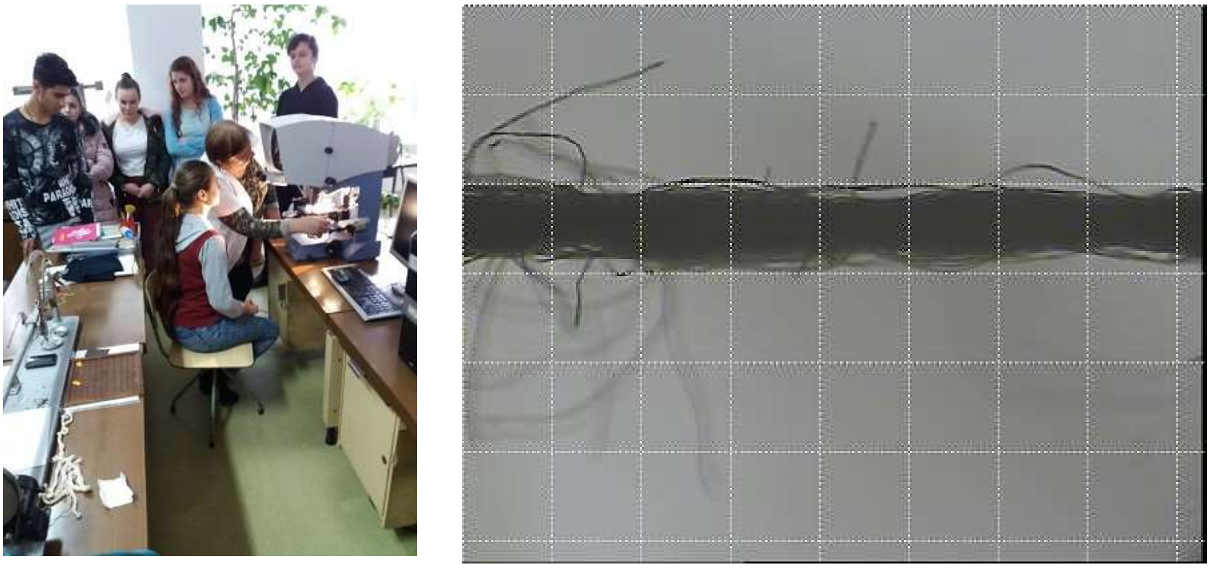
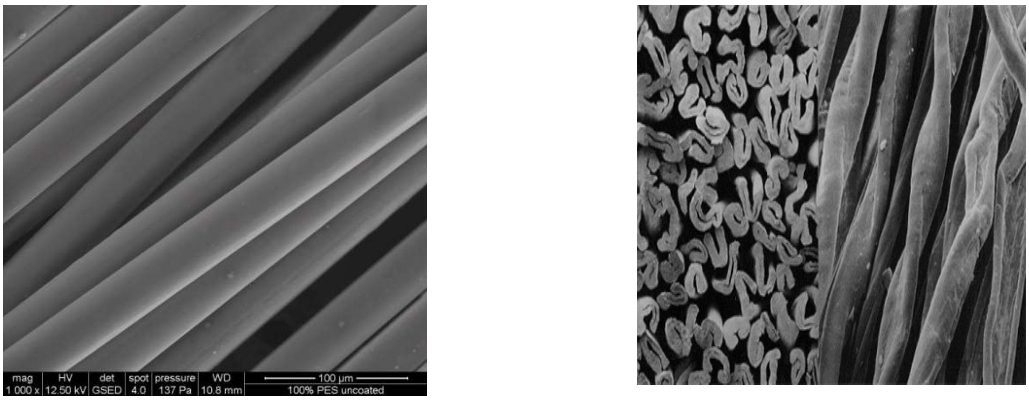


Fig. 1 a) and b) – Optical microscope for determination of yarn diameter and image of the yarn

Scanning Electronic Microscope (SEM) allows magnification of the studied sample of 1000x up to 8000x. Thus, a more detailed view of the fiber length and even a detailed view of the fiber's cross section is possible (Fig. 2 a) and b)).



a) Polyester fibres b) Cotton fibres

Fig. 2 SEM Images of the fibres

Various geometries of the cross-section are possible, depending on the raw material of the fiber / yarn. Figure 3 presents the geometries of the fiber's cross-section for most used natural and made-made fibers, such as: wool, natural silk, raw silk, cotton, mercerized cotton, flax, viscose, polyester, polyamide, trilobite polyamides; wet-spinning acryl; dry-spinning acryl, mod acryl, bi-component fibres.

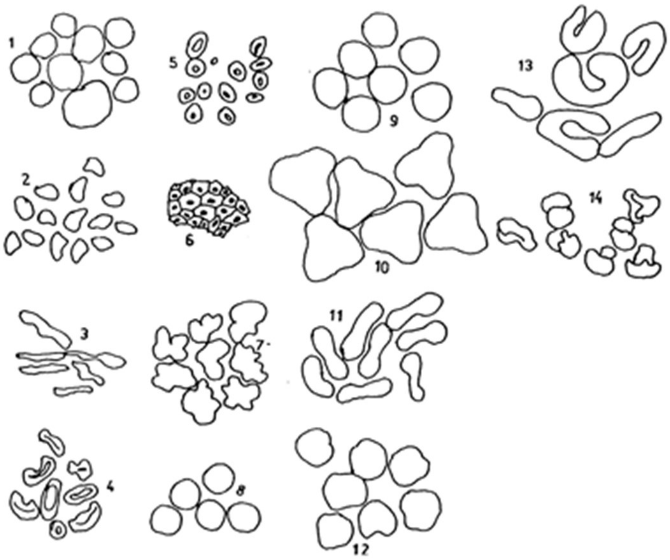


Fig. 3 – Cross section of the fibres [2]
1- wool; 2- natural silk; 3- raw silk; 4-cotton; 5- mercerized cotton; 6- flax; 7- viscose; 8- polyester; 9-polyamides; 10 – trilobite polyamides; 11- wet-spinning acryl; 12- dry- spinning acryl; 13-modacryl; 14 – bi-component fibres.

1.3 Hygroscopicity

Hygroscopicity is fiber's property to retain and to retrocede (release back) in the atmosphere water vapors. Fiber hygroscopicity depends of a multitude of factors, such as: polymer chemical composition, supramolecular structure, additional substances content. The water quantity within fibers at a certain moment depends on their hygroscopicity and the parameters of environment where they are kept [1].

Humidity quantity retained by fibers in standard microclimate conditions (temperature of 20^{o}C and air humidity of 65%) are named moisture content or regain. There are several domains which set regulations regarding fiber hygroscopicity. Thus, hygroscopic fibers are recommended for body wear, to impart adequate hygienic-functional properties to products. Fibers with regain of 6% are recommended for clothing products, while fibers with zero regain are recommended for technical textiles used as electrical isolators.

Water content within fibers changes a lot of their properties, such as: mass, dimensions, mechanical properties, electrical properties etc. For this reason, content of humidity is considered at quantitative uptake, while investigation of various properties is made after preliminary conditioning of fibers, yarns or other textile materials (textiles are kept for 24-72 hours in standard climate conditions).

Humidity of textile fibers and yarns: water content of a textile material is determined in prescribed conditions and expressed in percent.

Moisture regain (also called just regain) is the percentage of water a fiber or yarn can hold relative to the dry weight of the fiber or yarn in a particular environment, which is usually standardized at 20°C ± 2°C, 65% ± 4% relative humidity [1].

$$R = \frac{m_i - m_u}{m_u} \times 100(\%) \qquad (8)$$

Where:
m_i = sample mass before drying, in grams
m_u = sample mass after drying, in grams

1.4 Fiber length

From point of view of length, fibers are classified within:

- Fibers with undefined length
- Fibers with defined length

Fibers with undefined length, named filaments, are considered those natural or chemical fibers which length is depending on the size of feeding format (cocoon in case of natural silk, bobbin in case of chemical filamentary yarns etc.) [8].

Fibers with defined length include three main categories:

- Short fibers – fibers with length of up to 60 mm; this category includes cotton fibers, cotton type chemical fibers, flax tow; such fibers are processed by specific equipment for cotton spinning;
- Medium fibers – fibers with length between 60 mm - 150 mm; these are animal hairs and chemical fibers type wool; such fibers are processed by specific equipment for wool spinning;
- Long fibers – fibers with length up to 150 mm, such as technical bast fibers, horses hair etc., such fibers are processed on specific equipment for bast fiber spinning.

For most of the fibers, length is evaluated by mean values, variation coefficient and short fiber percentage content [1].

The content of fibers with multiple length are computed with formula:

$$\text{Fibers with multiple lengths} = \frac{m_1}{m_2} \times 100(\%) \qquad (9)$$

Where:
m_1 – fiber mass with multiple length, [g];
m_2 – fiber mass selected for determination, [g].

Determination of fibers length: mean length of fibers L_s is computed as weighted average according to relation:

$$L_s = \frac{m_1 L_1 + m_2 L_2 + \cdots m_n L_n}{m_1 + m_2 + \cdots + m_n} \quad \text{(mm)} \qquad (10)$$

Where:

$m_1, m_2 \ldots m_n$ = total mass of fibers belonging of a length class in milligrams [mg]

$L_1, L_2 \ldots L_n$ = average length of fibers of respective length class, in millimeters [mm]

1.5 Textile yarns twist

Twist represents the number of rotations of yarn around its axis, related to the yarn's length. Twist is preferentially expressed in twists per meter, but may be expressed in twists per centimeter, too [9]. Figure 4 presents the testing procedure for determination of yarn's twist on the Twist tester apparatus.

Fig. 4 – Twist Tester

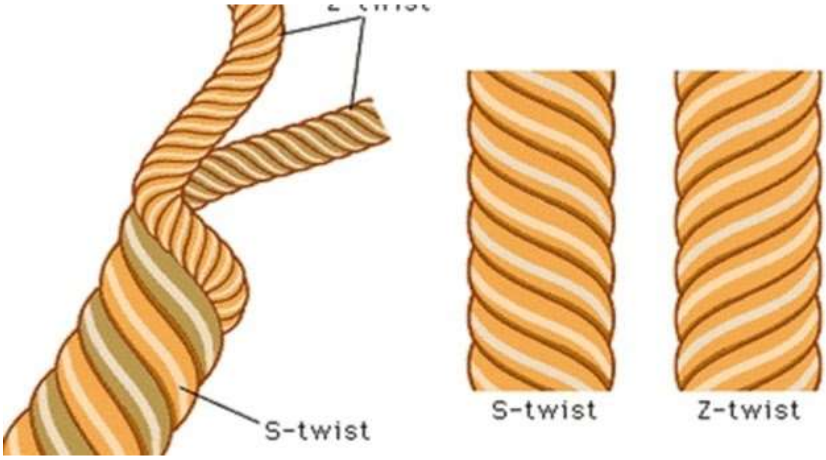


Fig. 5 – Yarns Twist [20]

There are two types of yarn twist S twist and Z twist, Figure 5 presents the two types of possible twists.

“S” TWIST: A single yarn has “S” twist if when it is held in the vertical direction, the fibers inclined to the conform in direction of slope of the contact portion of the letter “S” axis of the yarn.
“Z” TWIST. A single yarn has “Z” twist if when it is held in the vertical direction, the fibers inclined to the yarn axis conform in the direction of the slope to the central portion of the letter “Z”.
In the designation of yarns, it is essential to specify the direction of twist. Besides its importance in simplifying the trade, it is of great technical importance in designing fabrics. For example, in a twill fabric (a type of weave, see Chapter 2), the direction of twist in the yarn is of particular importance in determining the predominance of twill effect. For a right-handed twill, the best contrasting effect will be obtained when a yarn with Z twist is used; on the other-hand a left-handed twist will produce a fabric having flat appearance.
These direction of twist in a yarn have different naming that can be used:
- right-handed Twist. S twist or clock-wise and
- left-handed Twist. Z twist or anticlockwise.

Single yarns are generally Z twisted in anticlockwise direction, whereas the S or right-handed twist is very common in ply yarns [10]. Ply yarns are made out of twisting two, three or more single yarns. Computing and expressing of results is done according to standard EN ISO 2061- the average twist is computed in twists per meter [1], using the relation:

$$t_x = \frac{1000}{l} x \qquad (11)$$

Where:
t_x = twist, in twists/m
l – length before untwisting
x – total number of twists

1.6 Tensile strength

Definitions

- Breaking force (weight): maximal force applied to a sample in a tensile test for producing break. For yarns, tensile strength or weight is expressed in centi-newton (cN);
- Elongation at break: increase of sample’s length related to tensile force. For yarns, tensile strength is expressed as percent out of initial length;
- Breaking tenacity: ratio between sample’s breaking force and linear density; usually expressed in cN/tex;
- Clamp: component of tensile testing machine used for fixing the sample with appropriate gears;
- Distance between clamps: length of sample measured between the points of fastening in tensioned state;
- Tensile testing machine with constant speed of sample’s elongation (CRE): testing machine with a fixed clamp holding one end of the sample and a

second mobile clamp moving away of the fixed clamp with constant speed holding the other end of the sample.

- Yarn length presented in an adequate form for using, manipulating, storing etc. Formats may have support (such as coins, bobbins) or not (bundles, clusters).

Computing and expressing measurement results are performed by means of electronic dynamometer:

- **Breaking force**
  The average breaking force is computed, *F* in centi-newton, with one decimal precision;
- **Elongation at break**
  The average breaking elongation is computed, ε, in percent, with one decimal precision.

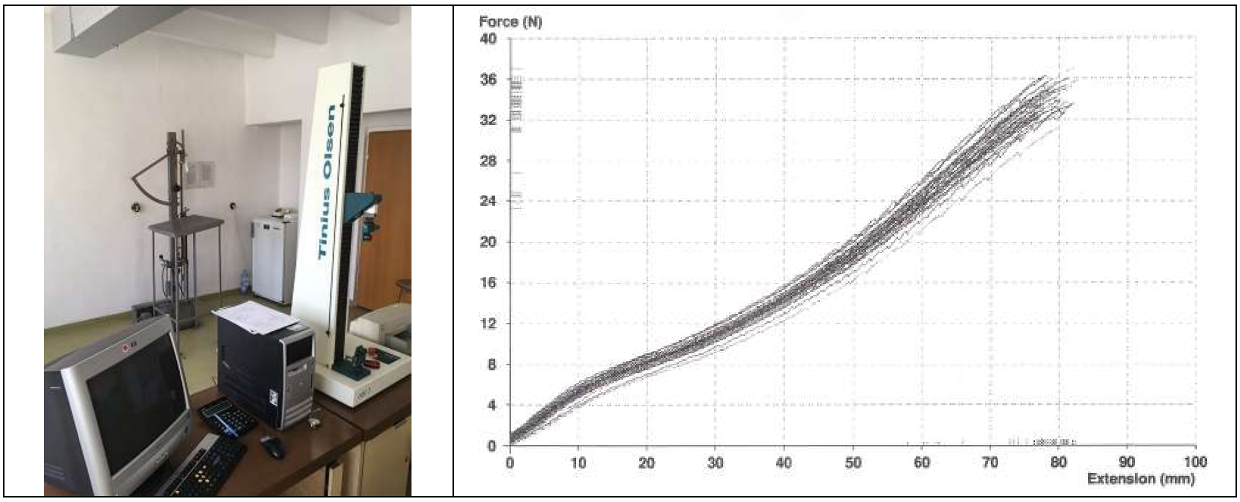


Fig. 6 a) and b) – Dynamometer and stress / strain diagram

Elongation at break ε is computed as:

$$\varepsilon = \frac{\Delta\, l}{l_0} \qquad (12)$$

Where:
l_0= initial length of fiber / yarn [mm]
$\Delta l = l - l_0$ = length modification during tensile stress [mm]

Example and exercise 1 shows a table generated by the dynamometer for testing tensile strength and elongation at break of a silver yarn with linear density of 30 Tex and 500,0 mm length. The fifth column length modification during tensile stress Δl is computed out of relation (12), taking into account that $l_0 = 500\, mm$. Please compute length modification for the next six values. The stress-strain diagram of figure 6b) shows on the x-Axis the length modification [mm] and on the y-Axis the force [N]. It presents the relation between length modification and tensile stress.

Table 2 – Tensile strength and elongation at break of the silver yarn

No.	Linear density T_{tex}[tex]	Tensile strength *F* [N]	Elongation ε [%]	Length modification Δl [mm]
1	30,00	14,00	26,58	132,90
2	30,00	13,68	25,80	129,00
3	30,00	13,50	25,98	129,90
4	30,00	13,00	25,17	125,85
5	30,00	12,93	24,60	123,00
6	30,00	13,32	25,38	126,90
7	30,00	12,75	24,17	?
8	30,00	12,07	23,17	
9	30,00	13,00	24,60	
10	30,00	12,07	23,00	
11	30,00	12,75	24,38	
12	30,00	12,25	23.60	

1.7 Young's modulus

Young's modulus is calculated by drawing a tangent to the initial linear portion of the stress strain curve, selecting any point on this tangent, and dividing the tensile stress σ by the corresponding elongation ε. For purposes of this calculation, the tensile stress shall be calculated by dividing the load *F* by the average original cross section of the test specimen *A*. The result is expressed in Pascal [Pa] and reported to three significant figures. Most materials have a (at least small) domain of linearity between stress and elongation, which is called Hooke's domain. On this domain following equation applies:

$$E = \frac{\sigma}{\varepsilon} \quad (13)$$

Where:

$$\sigma = \frac{F}{A} \quad (14)$$

And ε is computed according to equation (12).
F = breaking force on the Hook domain.
A = mean value of transversal cross-section of the fibers or yarns.
σ = the tensile stress
E = Young's modulus

Example 2 is on how to compute the Young's modulus for the silver yarn, knowing that it has a diameter of D = 0.280 mm, measured with the optical microscope (fig.1):

- First we compute the cross-section of the yarn $A = \frac{\pi D^2}{4} = 0{,}0615\, mm^2$
- Then we compute the tensile stress for an average value of six first values *F*= 13.41 N, $\sigma = \frac{F}{A} = 218 \frac{N}{mm^2}$

- Finally, we take an average value of $\varepsilon = 0.26$ and compute the Young's modulus $E = \frac{\sigma}{\varepsilon} = 56.68 \frac{N}{mm^2}$

Exercise 2: please compute the Young's modulus for the next six values.

1.8 Tenacity

Breaking tenacity (T) is computed with relation [11]:

$$T = \frac{F}{T_{tex}} \; [cN/tex] \tag{15}$$

Where:
F – the average breaking force, in Newton [N].
T_{tex} – average linear density, in [tex].

Table 3 – Tenacity of the silver yarn

No.	Linear density T_{tex}[tex]	Tensile strength *F* [N]	Tenacity [N/tex]
1	30,00	14,00	0,4667
2	30,00	13,68	0,4558
3	30,00	13,50	0,4500
4	30,00	13,00	0,4333
5	30,00	12,93	0,4308
6	30,00	13,32	0,4442
7	30,00	12,75	?
8	30,00	12,07	
9	30,00	13,00	
10	30,00	12,07	
11	30,00	12,75	
12	30,00	12,25	

Exercise 3: Please compute the tenacity for the next six values.

2. The modality to introduce physical quantities

Mathematical relation for physical property	Physical meaning	Mathematical meaning
$T_{tex} = \frac{M\,[g]}{L\,[km]}$	Linear density is a measure for the fineness of a fiber/yarn and may be expressed as product between cross-section area and specific mass	Ratio between two physical quantities in the SI system: mass [kg] and length [m]. Product between cross-section area [m^2] and specific mass [kg/m^3].
$N_m = \frac{L\,(m)}{M\,(g)} = \frac{L}{V \cdot \rho} = \frac{L}{A \cdot L \cdot \rho} = \frac{1}{A \cdot \rho}$		

$R = \frac{mi - mu}{mu} \times 100(\%)$	Regain expresses the uptake of water vapors within fiber/yarn volume.	Difference of masses [kg] expressed ratio and percent [%]
$Ls = \frac{m1L1 + m2L2 + \cdots mnLn}{m1 + m2 + \cdots + mn}$	Fiber length [m] represents a weighted average between mass [kg] and length [m] of various component fibers.	Weighted average
$tx = \frac{1000}{l}x$	Twist represents the number of rotations of yarn around its axis.	Ratio between number of twists [1] and yarn's length [m].
$\sigma = \frac{F}{A}$	Tensile stress σ represents the force applied on the unit surface and has the significance of pressure.	Ratio between the tensile strength [N] and the unit surface [mm^2]. It results the unit for pressure Pascal [Pa].
$\varepsilon = \frac{\Delta l}{l_0}$	Elongation at break ε represents the relative elongation to the initial length.	Ratio between two lengths (one absolute one relative). The ratio has the unit [1].
$E = \frac{\sigma}{\varepsilon}$	Young modulus shows the stiffness of a textile material – fiber, yarn.	Ratio between the tensile stress [Pa] and the breaking elongation [1].
$T = \frac{F}{Ttex}$	Tenacity represents the customary measure of strength of a fiber/ yarn.	Ratio between the breaking force [F] and the linear density [N_t].

3. The meaning of physical quantities for smart textiles materials

Physical property fiber/ yarn	Application for smart textiles
Linear density	Conventional and conductive yarns within a smart fabric should have similar linear density. When choosing an electrical conductive yarn out of stainless steel or silver for inserting in the textile structure (weaving, knitting etc.), it should have the same linear density as the conventional yarn out of cotton, polyester etc.
Hygroscopicity	Hygroscopicity indicates the potential of fibers and yarns to absorb water vapors. Electric insulating properties of fibers and yarns are changed with uptake of water vapors, for water is electrically conductive. Thus, conventional yarns within a smart fabric, should be selected with low values for hygroscopicity.

Fiber length	Fibers length is very important for the main characteristics of spun yarns and textile materials: tensile strength, elongation, surface appearance, comfort properties etc. It is an decisive indicator for the smart textile performances.
Twist	The twist of the yarn is in correlation with fiber length, type of the fibers, technological process and field of final use of the textile materials. It is different for smart woven fabric against knitting ones and has high influence on the tensile strength, elongation, surface appearance.
Elongation at break	Very important for the stability of smart textiles and their lifetime. Maintaining shape and size stability especially for smart textiles are crucial to maintaining their performance.
Young's modulus	Young's modulus has the same importance as breaking elongation and offer the possibility to choose the best raw materials for smart textiles in order to obtain and maintain maximum performances.
Tensile strength and relative elongation	Tensile strength and relative elongation show us the mechanical resistance of a fiber or yarn. This is important in some smart textile applications, for instance Personal Protection Equipment for outdoor conditions, electromagnetic shields or tents.
Tenacity	Depending on final application of smart textile, either a clothing application or a technical application, various properties parameters should be used: For smart clothing comfort properties are most important: lightweight, air permeability, resistance to water vapors and thermal resistance. For technical textiles most important properties are: tenacity, hydrophobic character etc.

Conclusions

Figure 7 showcases a smart textile end-product as final application of fibers and yarns.

Fig. 7 – Smart textile product with LEDs [12]

Conductive yarns are used for integration of sensors and other electronic devices with textile fabrics through weaving, knitting, braiding or embroidery processes. In the lifetime of the textile also several washing cycles might occur [11].

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